Biospherical Instruments Inc.

APPLICATION NOTE:

Light Quantities, Units, and Conversion Factors

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Light Quantities, Units, and Conversion Factors

"Light" is defined as electromagnetic radiation that is visible to the human eye. Because of the many effects that light has on living and non-living matter, different units are used to quantify the intensity of light. Most common are radiometric units, which characterize electromagnetic radiation in terms of energy; quantum units, which take into account that radiation is made up of discrete parcels of energy called photons or quanta; and photometric units, which quantify radiation in terms of its perceived brightness to the human eye. The objective of this document is to explain the different ways to quantify radiation, their quantities and units, and to provide factors for converting units of the different systems. The discussion focuses on quantities and units used by radiometers available from Biospherical Instruments. The narrative is kept simple to be useful for people new to the field of radiometry. More in-depth information is provided in footnotes and in the works by Morel and Smith (1974 and 1982), and Kirk (2011).

Table 1 gives an overview of radiometric and photometric quantities, their symbols, and units used in this document. All elements are explained in more detail below.

1. RADIOMETRIC QUANTITIES AND UNITS

Radiometric units quantify the energy of radiation. The SI[1,2] unit for "radiant energy" (symbol Q) is the Joule[3] (J). A closely related radiometric quantity is "radiant flux"[4] (symbol Φ), which is the radiant energy per unit time and is in units of the Watt (symbol W). Depending on the geometry of the measurements tasks, additional radiometric quantities need to be introduced, resulting in different units of the measurement result. Most common radiometric quantities are "radiance," "irradiance," and "scalar irradiance:

- **Radiance** (symbol L) quantifies the brightness of an object. It is defined[5] as radiant flux Φ arriving per unit area A (the SI unit of area is square meter or m^2) and per unit solid angle[6] $d\Omega$ (the unit of solid angle is steradian or sr) on a flat receiver oriented perpendicular to the source of radiation[7,8];

$$L = \frac{\Phi}{A\Omega}. \tag{1}$$

The SI unit for radiance is [W m^{-2} sr^{-1}]. It can be difficult for a novice to grasp "radiance." A good introduction can be found in the Wikipedia page on radiance: *http://en.wikipedia.org/wiki/Radiance*.

- **Irradiance** (symbol E) quantifies radiant flux per unit area of a flat surface[9]:

$$E = \frac{\Phi}{A}. \tag{2}$$

1 "SI" refers to the International System of Units, see *http://en.wikipedia.org/wiki/International_System_of_Units.*
2 References to Wikipedia articles are given for the convenience of our customers. Care should be exercised when reading them, however, because Wikipedia is not refereed or peer-reviewed. BSI does not endorse nor take any responsibility for information contained in these articles.
3 The joule is the unit for all forms of energy, not just radiant energy.
4 Radiant flux may also be called radiant power.
5 This description defines radiance from the standpoint of a receiver or detector. Radiance can also be defined from the standpoint of a source such as a bright wall. In this case, radiance becomes the radiant flux emitted per area of the wall and per solid angle.
6 "Solid angle" is defined as the area of the segment of a sphere with unit radius (see the Wikipedia article *http://en.wikipedia.org/wiki/Solid_angle*). The area of an entire sphere with unit radius is 4π. Hence, a solid angle of 4π indicates a complete sphere, and a solid angle of 2π indicates half a sphere or a hemisphere. Radiometers that are sensitive to radiation from only one hemisphere are therefore called "2π-sensors."
7 If the receiver is not oriented normal to the source, radiance is defined as $L = \Phi / [A\Omega\cos(\theta)]$ where θ is the angle between the surface normal and the direction of the incident radiation.
8 More accurately defined, radiance is the radiant flux $d\Phi$ incident on an infinitesimal element of a surface and originating from an infinitesimally small solid angle around the direction of incidence, divided by the area dA of the surface element, the size of the solid angle $d\Omega$, and the cosine of the angle θ between the surface normal and the direction of incidence: $L = d2\Phi / [dA\, d\Omega \cos(\theta)]$. Throughout this document, we assume that the radiant flux is constant over the surfaces and solid angles considered. In this case, the definition of radiance simplifies to $L = \Phi / [A\Omega\cos(\theta)]$.
9 More accurately defined, irradiance is the radiant flux $d\Phi$ incident on an infinitesimal element of a surface divided by the area dA of that element: $E = d\Phi / dA$. As in the case of radiance, we assume that the radiant flux is constant over the surface. In this case, the definition of irradiance simplifies to $E = \Phi / A$.

Table 1. Comparison of radiometric and photometric quantities, and their SI units.

RADIOMETRIC QUANTITIES			PHOTOMETRIC QUANTITIES		
Quantity	*Symbol*	*SI Unit*	*Quantity*	*Symbol*	*SI Unit*
Radiant energy	Q	Joule [J]	Luminous energy	Q_v	lumen second [lm s]
Radiant flux	Φ	Watt [W]	Luminous flux	Φ_v	lumen [lm]
Radiant intensity	I	Watt per steradian [W sr^{-1}]	Luminous intensity	I_v	candela [cd]
Radiance	L	Watt per steradian per square meter [W sr^{-1} m^{-2}]	Luminance	L_v	candela per square meter [cd m^{-2}]
Irradiance	E_d	Watt per square meter [W m^{-2}]	Illuminance	E_v	lux [lx]
Scalar irradiance	E_0	Watt per square meter [W m^{-2}]	Scalar illuminance	E_{0v}	lux [lx]

The SI unit for irradiance is [W m^{-2}]. Irradiance E can be calculated from radiance L by integrating over the solid angle subtended by the hemisphere above the surface:

$$E = \int_{(2\pi)} L(\theta,\varphi)\cos(\theta)d\Omega = \int_{\theta=0}^{\pi/2}\int_{\varphi=0}^{2\pi} L(\theta,\varphi)\cos(\theta)\sin(\theta)d\theta d\varphi. \tag{3}$$

In Eq. (3), θ and φ are spherical coordinates, specifically θ is the "zenith angle," describing the angle between the normal of object's surface and the direction of the incident radiation, and φ is the azimuth angle. It can be shown that $d\Omega$ is equal to $\sin(\theta)d\theta d\varphi$. The term 2π indicates integration over the hemisphere "seen" by the object (e.g., the entire sky—from horizon to horizon—for an "irradiance radiometer" deployed outdoors). Note that the equation includes the term cos(θ). Because of this term, irradiance depends on the direction of incoming radiation. Phrased differently, irradiance has a unique value for every orientation of the receiving object in space.

Scalar irradiance is abbreviated with the symbol E_0 and can be visualized as the radiant flux absorbed by a small sphere from all directions divided by the surface area of this sphere. This quantity has a single unique value at each point in space as contrasted to irradiance defined above. Scalar irradiance is useful to quantify effects that do not depend on direction, such as effects of radiation on cells suspended in water or on air molecules. The units of scalar irradiance is Watts per square meter (W m^{-2}), which is the same unit as irradiance. Scalar irradiance, E_0, is calculated by integrating radiance L over the entire sphere (solid angle 4π):

$$E_0 = \int_{(4\pi)} L(\theta,\varphi)d\Omega = \int_{\theta=0}^{\pi}\int_{\varphi=0}^{2\pi} L(\theta,\varphi)\sin(\theta,\varphi)d\theta d\varphi. \tag{4}$$

Integration of L over the upper hemisphere only results in the "downward scalar irradiance" (symbol E_{0d}); integration over the lower hemisphere gives the "upward scalar irradiance" (symbol E_{0u}); E_0 is the sum of both components: $E_0 = E_{0d} + E_{0u}$.

In the atmospheric research communities, "scalar irradiance" is called "actinic flux." Less common additional terms for this quantity are "actinic flux density," "actinic irradiance," and "spheradiance." Scalar irradiance is the preferred term according to the Committee on Radiant Energy in the Sea of the International Association of Physical Oceanography (Morel and Smith 1982).

As with all metric units, prefixes (e.g., m for milli, µ for micro, c for centi) can be used as multipliers. For example, Biospherical Instruments sometimes uses the unit [mW cm^{-2} = 10 W m^{-2}] to quantify irradiance measurements.

All quantities and units discussed above characterize the energy of radiation received over the entire spectral range of electromagnetic radiation. For spectral characterization of radiation, wavelength-dependent quantities have to be introduced. For example, "spectral radiance," $L(\lambda)$, is defined as

$$L(\lambda) = \frac{dL}{d\lambda}, \tag{5}$$

where dL is the radiance produced by photons with wavelengths within the infinitesimally small wavelength interval $d\lambda$. Similarly, "spectral irradiance," denoted $E(\lambda)$, is defined as $E(\lambda)=dE/d\lambda$, and "spectral scalar irradiance," denoted $E_0(\lambda)$, is defined as $E_0(\lambda) = dE_0/d\lambda$. The "generic" unit of $E(\lambda)$ and $E_0(\lambda)$ is [$\mathrm{W\,m^{-2}\,nm^{-1}}$], where nm is nanometers (1 nm = 10^{-9} m). Biospherical Instruments often uses the related unit [$\mathrm{\mu W\,cm^{-2}\,nm^{-1}}$]. With this definition, irradiance can be expressed as

$$E = \int_{\lambda=\lambda_1}^{\lambda_2} E(\lambda)d\lambda. \tag{6}$$

Note that the wavelength integration is between the wavelengths λ_1 and λ_2.

2. QUANTUM UNITS

Quantum units take into account that radiation is made up of discrete parcels of energy called photons or quanta. Conversion of radiometric units to quantum units is wavelength dependent because the energy of a photon, Q_p, is determined by its wavelength:

$$Q_p = \frac{hc}{\lambda}, \tag{7}$$

where h is the Planck constant ($h = 6.626 \times 10^{-34}$ Js), c is the speed of light ($c = 2.997 \times 10^{-8}$ Js = $\mathrm{ms^{-1}}$), and λ is the wavelength of the photon. The number of quanta associated with radiant energy Q can be simply calculated by

$$\text{quanta} = \frac{Q}{Q_p} = \frac{Qz}{hc}. \tag{8}$$

If Q is provided in joules and wavelength is given in nanometers, the following conversion formulas apply:

$$\text{quanta} = 5.043 \times 10^{15} Q\lambda, \tag{9a}$$

and

$$Q = 1.986 \times 10^{-16} \frac{\text{quanta}}{\lambda}. \tag{9b}$$

Note that these formulas' factors have to be modified if radiant energy is provided in a unit other than joules.

In the biological communities, the unit einstein is often used to denote one mole of quanta (or photons): 1 Ein = 6.022×10^{23} quanta, where 6.022×10^{23} is the Avogadro constant.

Quantum units have to be used to quantify the amount of light available for photosynthesis. This is necessary because photosynthesis is mostly driven by the number of photons absorbed by a cell rather than the total amount of energy contained by these photons. Only photons with wavelengths from 400–700 nm can be used for photosynthesis. The term "Photosynthetically Active Radiation"[10] (PAR) has, therefore, been coined for the wavelength range of 400–700 nm.

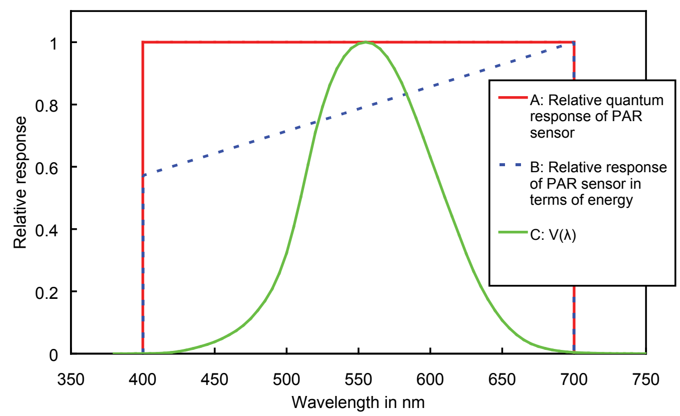

Figure 1. Comparison of the relative response of a PAR sensor in terms of sensitivity to photons (A) and in terms of sensitivity to energy (B). Data set C is the "photopic luminosity function" $V(\lambda)$ used for the calculation of photometric quantities (Section 3).

Data set A in Fig. 1, shows the ideal relative response of a PAR sensor in terms of sensitivity to photons: the sensor has no sensitivity below 400 nm or above 700 nm, and a constant sensitivity from 400–700 nm. When the responsivity of the same sensor is plotted with respect to its sensitivity to the energy content of these photons, the function has an increasing trend with a value of 400/700 = 0.571 at 400 nm, and 1 at 700 nm (Data set B in Fig. 1). The reason for the lower responsivity at 400 nm can be easily explained: photons at 400 nm are more energetic than photons at 700 nm, Eq. (7). To get the same number of photons at 400 nm than at 700 (i.e., to get the desired constant quantum response), the source radiation has to be more energetic at the shorter wavelength. For example,

10 The term "Photosynthetically Available Radiation" is sometimes used instead of Photosynthetically Active Radiation (PAR). The term PAR has been applied to both energy and quantum fluxes. The most common use today refers to total quantum flux between 400 and 700 nm.

the spectral irradiance of a source needs to be larger by a factor of 700/400 = 1.75 at 400 nm than for 700 nm. As a consequence, a PAR sensor has to be less sensitive in terms of energy at the shorter wavelength such that the product of spectral irradiance and sensor responsivity is constant. This is achieved with the function shown in Data set B. The spectral response of PAR sensors offered by Biospherical Instruments mimic this function.

Morel and Smith (1974) studied the relationship between total quanta and total energy for aquatic photosynthesis and concluded that 1 W of PAR corresponds to approximately 2.77×10^{18} quanta per second for marine atmospheres above the water surface with Sun altitudes above 22° (solar zenith angle smaller than 68°). This "rule of thumb" is helpful to provide a rough estimate of PAR if only measurements in terms of energy are available. Using a PAR sensor with the correct spectral response is, therefore, more accurate.

Note that PAR does not specify the measurement geometry; it can be used both in context of a flat receiver (cosine PAR sensors) and a spherical receiver (scalar PAR sensor):

- Cosine PAR Sensors (flat receiver)

If PAR is referenced to a flat receiver, the quantity "quantum irradiance" or "Photosynthetic Photon Flux Density"[11] (PPFD) is used. It is defined as number of photons in the 400–700 nm wavelength interval arriving per unit time on a unit area of the flat receiver. PPFD can be calculated from spectral irradiance using Eq. (6):

$$\mathrm{PPFD} = \int_{\lambda=400}^{700} E(\lambda)\frac{\lambda}{hc}\,d\lambda. \tag{10}$$

PPFD is typically quantified in the units of [µEin cm^{-2} s^{-1}] or [µEin m^{-2} s^{-1}]. The QCP series of sensors available from Biospherical Instruments measure PPFD. These sensors are equipped with "cosine irradiance" collectors and are, therefore, referred to as "Cosine PAR Sensors." Alternative terms for PPFD are "quantum flux density," or "PAR quantum irradiance."

- Scalar PAR Sensors (spherical receiver)

11 The term "quantum flux density" describes the number of photons moving per second through the unit area of a surface. "Photosynthetic photon flux density" is, therefore, the number of photons with wavelengths from 400–700 nm moving per second through this surface.

If PAR refers to a spherical receiver, the quantity "Quantum Scalar Irradiance" (QSI) is used. It is defined as the integral photon flux of photons in the 400–700 nm wavelength interval at a point in space from all directions around the point. It can be calculated from scalar irradiance, E_0, similar to Eq. (10):

$$Q(E_0) = \int_{\lambda=400}^{700} E_0(\lambda)\frac{\lambda}{hc}\,d\lambda. \tag{11}$$

QSI is also quantified in the units of [µEin cm^{-2} s^{-1}] or [µEin m^{-2} s^{-1}]. Biospherical Instruments offers several quantum scalar sensors, specifically the QSP and QSL series of instruments, as well as the Advanced Multi-purpOse Usb Radiometer (AMOUR) sensor. These sensors are equipped with spherical collectors made of Teflon® with an almost uniform directional response. Alternative terms for QSI are "quantum flux density," "photosynthetic photon flux fluence rate," and "photon flux fluence rate."

Figure 2 shows a comparison of quantum irradiance (i.e., PPFD) and quantum scalar irradiance calculated with a radiative transfer model for clear skies as a function of solar zenith angle. The surface albedo (ground reflectance) was set to 7% in these calculations and low atmospheric aerosol concentrations were assumed. The figure also shows about one year of measurements of a quantum irradiance sensor mounted on the roof platform of Biospherical Instruments.

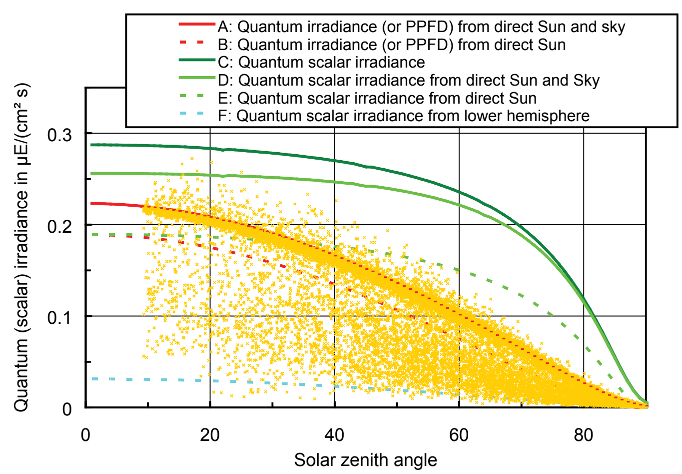

Figure 2. Comparison of quantum irradiance (red lines) and quantum scalar irradiance (green and blue lines) calculated with a radiative transfer model for clear skies. The orange dots show measurements of a quantum irradiance sensor mounted on the roof platform of Biospherical Instruments.

The high point-density in the upper part of the measured data set corresponds to clear sky conditions and agrees well with the model calculations (Data set A). Data set B is the modeled quantum irradiance originating only from photons of the direct solar beam, that is, no photons from sky light are included. Data set C is quantum scalar irradiance as defined in Eq. (11) resulting from photons from both the upper hemisphere (direct solar beam and sky) and lower hemisphere (photons reflected off the surface). Data sets D, E, and F represent quantum scalar irradiance originating from the upper hemisphere, direct Sun only, and lower hemisphere. Note that quantum scalar irradiance (Data set C) is always larger than quantum irradiance (Data set A). For overhead Sun (solar zenith angle of 0°), quantum irradiance and quantum scalar irradiance from the direct Sun are equal. Figure 2 gives an indication of the magnitudes of PPFD and QSI from different outdoor sources. As clouds and other factors have a large influence on the two quantities, measurements such as those provided by the Biospherical Instruments line of quantum irradiance and quantum scalar irradiance sensors are necessary to characterize the outdoor environment accurately.

3. PHOTOMETRIC QUANTITIES AND UNITS

Photometric quantities and units quantify radiation in terms of its perceived brightness to the human eye. Many radiometric quantities have a photometric analog as shown in Table 1. “Scalar illuminance” (the analog to “scalar irradiance”) is rarely used.

Any photometric quantity, X_v, can be calculated from the associated spectral radiometric $X(\lambda)$ quantity via the formula:

$$X_v = K_m \int_{\lambda=0}^{\infty} X(\lambda)V(\lambda)d\lambda. \tag{12}$$

For example, if $X(\lambda)$ is set to spectral irradiance $E(\lambda)$, Eq. (12) can be used to calculate illuminance $E(\lambda)$. The term $V(\lambda)$ in Eq. (12) is the “photopic luminosity function.” This function describes the relative wavelength-dependent sensitivity of the human eye to typical light levels occurring during the day and is shown in Fig. 2 (Data set C). The function is equal to one at 555 nm, which is the wavelength where the human eye is most sensitive. The term K_m is called the “maximum spectral luminous efficacy” and has the value 683 lm W^{-1}. If monochromatic light with a wavelength of 555 nm produces an irradiance of 1 W m^{-2}, the associated illuminance is, therefore, 683 lm m^{-2} = 683 lx.

An astonishing amount of additional units have historically been introduced for photometric quantities, and some are still in use. For example, illuminance can be quantified

Table 2. Conversion factors for monochromatic light.

WAVELENGTH	QUANTUM IRRADIANCE		IRRADIANCE†		ILLUMINANCE‡	
[nm]	[quanta cm^{-2} s^{-1}]	[µE cm^{-2} s^{-1}]	[W cm^{-2}]	[Ly min^{-1}]	[lx]	[fc]
350§	1×10^{17}	0.1661	0.0568	0.8139	0	0
400	1×10^{17}	0.1661	0.0497	0.7122	134	12
450	1×10^{17}	0.1661	0.0441	0.6330	11457	1064
500	1×10^{17}	0.1661	0.0397	0.5697	87646	8142
550	1×10^{17}	0.1661	0.0361	0.5179	245435	22801
600	1×10^{17}	0.1661	0.0331	0.4748	142684	13256
650	1×10^{17}	0.1661	0.0306	0.4383	22334	2075
700	1×10^{17}	0.1661	0.0284	0.4069	795	74
750§	1×10^{17}	0.1661	0.0265	0.3798	22	2

† The irradiance unit Ly min^{-1} is defined as “Langley per minute”; 1 Ly = 41840 J m^{-1}.

‡ The illuminance unit fc is defined as “foot-candle”; 2 fc = 10.764 lx.

§ These wavelengths are outside of the normal PAR range.

in the units of phot, nox, foot-candle, meter-candle, and flame. The conversion factors for these into SI units are:

1 nox = 0.001 lx

1 phot = 10,000 lx

1 foot-candle = 1 fc = 10.764 lx

1 meter-candle = 1 lx

1 flame = 43.055 lx

Similarly, luminance can also be quantified in the units of lambert, foot-lambert, nit, and stilb. The use of these units, however, is now discouraged; SI units should be used uniformly.

4. CONVERTING BETWEEN UNITS

For radiation quantities associated with polychromatic light sources, accurate conversion between radiometric, quantum, and photometric units is only possible if the spectral distribution of the light source is known *a priori,* or if it can be measured. This is a consequence of the integral formulas of Eqs. (10), (11), and (12). Conversion factors for monochromatic light can be readily calculated and are listed in Table 2. These factors depend strongly on wavelength for some quantities. Great caution has to be exercised if these factors are applied to broadband sources.

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